Evidence of longterm cyclic evolution of radio pulsar periods [*]

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Abstract

The measurements of pulsar frequency second derivatives have shown that they are $10^2 - 10^6$ times larger than expected for standard pulsar spin-down law. Moreover, the second derivatives as well as braking indices are even negative for about half the pulsars. We explain these paradoxical results on the basis of the statistical analysis of the rotational parameters ν, $\dot\nu$ and $\ddot\nu$ of the subset of 295 pulsars taken mostly from the ATNF database. We have found a strong correlation between $\ddot\nu$ and $\dot\nu$ for both $\ddot\nu > 0$ (correlation coefficient $r \approx 0.9$) and $\ddot\nu < 0$ ($r \approx 0.85$), as well as between ν and $\dot\nu$ ($r \approx 0.6 \div 0.7$). We interpret these dependencies as evolutionary ones due to $\dot\nu$ being nearly proportional to the pulsars' age.

The derived statistical relations as well as "anomalous" values of $\ddot\nu$ are well described by assuming the existence of long-time variations of the spin-down rate. The pulsar frequency evolution, therefore, consists of secular change of $\nu_{ev}(t)$, $\dot\nu_{ev}(t)$ and $\ddot\nu_{ev}(t)$ according to the power law with $n \approx 5$, the irregularities, observed within the timespan as timing noise, and the non-monotonous variations on the timescale of several tens of years, which is larger than that of the timespan. It is possible that the nature of long-term variations is similar to that of short-term ones. The idea of non-constant secular pulsars' braking index n is also analysed.

1. Introduction

The spin-down of radio pulsars is caused by the conversion of their rotation energy into emission. According to the "classical" approach, their rotational frequencies ν evolve obeying the spin-down law $\dot\nu = -K\nu^n$, where K is a positive constant that depends on the magnetic dipole moment and the moment of inertia of the neutron star, and n is the braking index. The latter can be determined observationally from measurements of ν, $\dot\nu$ and $\ddot\nu$ as $n = \nu\ddot\nu/\dot\nu^2$. For a simple vacuum dipole model of the pulsar magnetosphere $n = 3$; the pulsar wind decreases this value to $n = 1$; for multipole magnetic field $n \geq 5$ (Manchester & Taylor, 1977). At the same time the measurements of pulsar frequency second derivatives $\ddot\nu$ have shown that their values are much larger than expected for standard spin-down law and are even negative for about half of all pulsars. The corresponding braking indices range from -10^6 to 10^6 (D'Alessandro et al., 1993; Chukwude, 1993; Hobbs et al., 2004).

It was found that the significant correlations between $|\ddot\nu|$ ($|\ddot P|$) and $\dot\nu$ ($\dot P$) demonstrate the fact that the absolute values of the $\ddot\nu$ and $\ddot P$ are larger for younger (with large $|\dot\nu|$) pulsars

(Cordes & Downs, 1985; Arzoumanian et al., 1994; Lyne, 1999). The anomalously high and negative values of $\ddot{\nu}$ and n may be interpreted as a result of low-frequency components of the "timing noise" – a complex change of pulsars' rotational phase within a timespan (D'Alessandro et al., 1993). Or, as a result of any long-term influence on the pulsars' spin-down. (Gullahorn & Rankin, 1977; Demiańsky & Proszyński, 1979)

It is clear that the timespan of observations is by no means intrinsic to pulsar physics. Indeed, the variations of rotational parameters may take place on larger timescales as well. However, the timescale of observations naturally divides these variations into two separate classes of manifestations: (i) the well-known "timing noise" – the residuals in respect to the best fit for the timing solution and (ii) the "long term timing noise" – the systematic shift of the best fit coefficients (i.e. in the measured values of ν, $\dot{\nu}$, $\ddot{\nu}$) relative to some mean or expected value from the model.

Up to date we know nearly 200 pulsars for which the timespan of observations is greater than 20 years, and the values of their $\ddot{\nu}$ still turn out to be anomalously large (Hobbs et al., 2004).

For example, for the *PSR B1706-16* pulsar, variations of $\ddot{\nu}$ with an amplitude of 10^{-24} s^{-3} have been detected on a timescale of several years (see Fig.7 in Hobbs et al. (2004)), with the value of $\ddot{\nu}$ depending on the time interval selected. However, the fit over the entire 25 year timespan gives a value of $\ddot{\nu} = 3.8 \cdot 10^{-25}$ s^{-3} with a few percent accuracy (which leads to a braking index $\approx 2.7 \cdot 10^3$).

In the current work we provide observational evidence of the non-monotonous evolution of pulsars on timescales larger than the typical contemporary timespan of observations (tens of years), using the statistical analysis of the measured ν, $\dot{\nu}$ and $\ddot{\nu}$. We estimate the main parameters of such long-term variations and discuss their possible relation to the low-frequency terms of timing noise. We have also derived the parameters of pulsar secular spin-down. We plotted the $\nu - \dot{\nu}$ diagrams for 295 and for 1337 "ordinary" radiopulsars. The smaller subset consists of pulsars with measured $\ddot{\nu}$. The bigger one – of all the "ordinary" pulsars with measured ν and $\dot{\nu}$, taken from the ATNF database (Manchester et al., 2005). We have found a good correlation beetween these two parameters and determnined the mean slope of the $\dot{\nu} - \nu$ distribution, which is in agreement with $n \sim 5$.
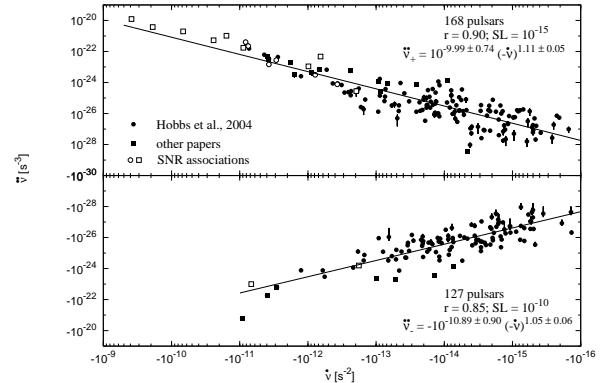

Fig. 1. The $\ddot{\nu} - \dot{\nu}$ diagram for 295 pulsars. The figure shows the pulsars taken from (Hobbs et al., 2004) as circles, and the objects measured by other groups as squares. Open symbols represent the relatively young pulsars associated with supernova remnants. Analytical fits for both positive and negative branches are shown as solid lines. Measurment errors are shown as error bars.

2. Statistical analysis of the ensemble of pulsars

As was stated above, earlier works have shown the possibility of long-term variations of pulsars' rotational frequency. Our statistical analysis shows that the measurements of most the $\ddot{\nu}$ reflect the pulsars' spin-down evolution on a timescale larger than the duration of observations, and uses the parameters of 295 pulsars. From the 389 objects of the ATNF catalogue (Manchester et al., 2005) with known $\ddot{\nu}$ we compiled a list of "ordinary" radio pulsars with $P > 20$ ms, $\dot{P} > 10^{-17}$ s/s, and with relative accuracy of second derivative measurements better than 75%. We excluded recycled, anomalous and binary pulsars. 26 supplementary pulsars from other sources (D'Alessandro et al., 1993; Chukwude, 1993) were added. The parameters of all pulsars were plotted on the $\ddot{\nu} - \dot{\nu}$ diagram (Fig. 1).

The basic result of the statistical analysis of this data is a significant correlation between $\ddot{\nu}$ and $\dot{\nu}$, both for 168 objects with $\ddot{\nu} > 0$ (correlation coefficient $r \approx 0.90$) and for 127 objects with $\ddot{\nu} < 0$ ($r \approx 0.85$). Both groups follow nearly linear laws, however they are not exactly symmetric relative to $\ddot{\nu} = 0$. We divided both branches into 6 intervals of $\dot{\nu}$, computed the mean values and their standard deviations of $\ddot{\nu}_{\pm}$ for each interval. We rejected the hypothesis of the branches symmetry with a 0.04 significance level. Also, the absolute values of $\ddot{\nu}_{+}$ are systematically larger than the corresponding $\ddot{\nu}_{-}$
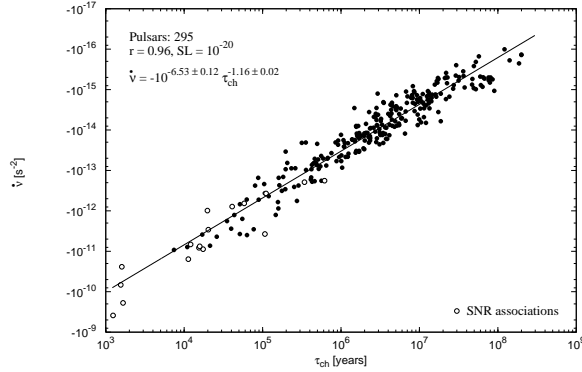
Fig. 2. The obvious $\dot{\nu} - \tau_{ch}$ dependence for pulsars with the measured second derivative. The open symbols represent pulsars associated with SNRs. The solid line represents the best fit, the dotted lines – 1-σ range. The index of power -1.16 ± 0.02 is significantly different from -1.0 which indicates the presence of the $\dot{\nu} - \nu$ correlation.
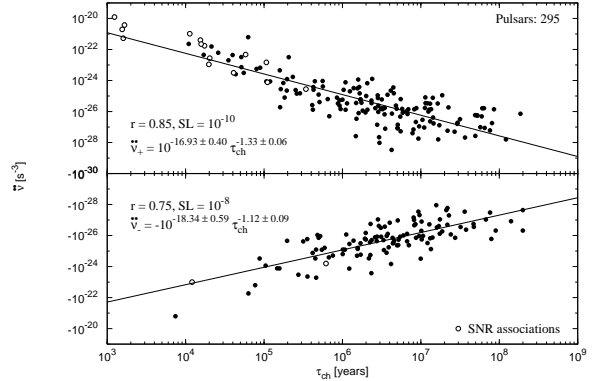

Fig. 3. The $\ddot{\nu} - \tau_{ch}$ dependence for pulsars with the measured second derivative. The open symbols represent pulsars associated with SNRs. This diagram is an argument in favour of the idea that pulsars evolve from large to small $|\ddot{\nu}|$ values independently from the $\ddot{\nu}$ sign.

(the difference is positive in 5 intervals out of 6) and the difference of analytical fits to branches is positive over the $-10^{-11} \div -10^{-15} \; s^{-2}$ interval of $\dot{\nu}$. These are the arguments in favour of a small positive assymetry of the branches.

We found an obvious correlation of $\dot{\nu}$ with the characteristic age $\tau_{ch} = -\frac{1}{2}\frac{\nu}{\dot{\nu}}$ ($r = 0.96$, see Fig. 2):

$$\dot{\nu} = -10^{-6.53 \pm 0.12}\tau_{ch}^{-1.16 \pm 0.02} \qquad (1)$$

This correlation will be discussed below in the light of the $\dot{\nu} - \nu$ dependency. The main feature of (1) is that the index of power -1.16 ± 0.02 is significantly different from -1.0. It is obvious that $\dot{\nu}$ would be $\sim \tau_{ch}^{-1}$ if ν and $\dot{\nu}$ were completely uncorrelated. Therefore, the derived result provides an argument in favour of the $\dot{\nu} - \nu$ correlation.

The $\dot{\nu}$ and τ_{ch} are nearly proportional, which leads to a significant correlation of τ_{ch} both with $\ddot{\nu}$ ($r = 0.85$ for the positive branch and $r = 0.75$ for the negative one, Fig. 3) and with n ($r = 0.75$ and $r = 0.76$ correspondingly, Fig. 8).

The correlations found are fully consistent with the results published in (Cordes & Downs, 1985; Arzoumanian et al., 1994; Lyne, 1999), as well as in (Urama et al., 2006). However, the branches with $\ddot{\nu} > 0$ and $\ddot{\nu} < 0$ in those works were not analysed separately from each other (not as $|\ddot{\nu}|$).

Young pulsars confidently associated with supernova remnants are systematically shifted to the left in Fig. 1 (open symbols). The order of magnitude of their physical ages roughly corresponds to that of their characteristic ages. This means that any dependence on $\dot{\nu}$ or τ_{ch} reflects the dependence on pulsar age. The $\ddot{\nu} - \dot{\nu}$ diagram (Fig. 1) may be interpreted as an evolutionary one. In other words, each pulsar during its evolution moves along the branches of this diagram while increasing the value of its $\dot{\nu}$ (which corresponds to the increase of its characteristic age). However, there is an obvious contradiction: on the negative branch, $\dot{\nu}$, being negative, may only decrease with time (since $\ddot{\nu}$ is formally the derivative of $\dot{\nu}$), and the motion along the negative branch may only be backward! This contradiction is easily solved by assuming non-monotonous behaviour of $\ddot{\nu}(t)$, which has an irregular component ($\delta\ddot{\nu}$) along with the monotonous one ($\ddot{\nu}_{ev}$), where the subscript "ev" marks the evolutionary value.

The characteristic timescale T of such variations must be much shorter than the pulsar life time and at the same time much larger than the timescale of the observations. As it evolves, a pulsar repeatedly changes sign of $\ddot{\nu}$, in a spiral-like motion from branch to branch, and spends roughly half its lifetime on each one. The asymmetry of the branches reflects the positive sign of $\ddot{\nu}_{ev}(t)$, and therefore, secular increase of $\dot{\nu}_{ev}(t)$ (i.e. all pulsars in their secular evolution move to the right on the $\ddot{\nu} - \dot{\nu}$ diagram). The systematic decreasing of the branches separation reflects the decreasing of the variations amplitude and/or the increasing of its characteristic timescale.

Any well known non-monotonous variations of $\dot{\nu}(t)$, like glitches, microglitches, timing noise or precession, will manifest themselves in a similar way on the $\ddot{\nu} - \dot{\nu}$ diagram and lead to extremely high values of $\ddot{\nu}$ (Shemar & Lyne, 1996; Stairs et al., 2000). However, their characteristic timescales vary from

weeks to years, and they are detected immediately. But here the variations on much larger timescales are discussed, and their study is possible only statistically, assuming the ergodic behaviour of the ensemble of pulsars.

In addition, the apparent branches separation on the $\ddot{\nu}-\dot{\nu}$ diagram is only due to the logarithmic scale of the plot. The spread of $\ddot{\nu}$ values in each branch reaches 3 orders of magnitude, i.e. the pulsars on the diagram cover almost fully the range of possible $\ddot{\nu}$ values (for each value of $\dot{\nu}$). Moreover, we observe the lack of pulsars near $\ddot{\nu} \sim 0$ because (i) the present accuracy of the $\ddot{\nu}$ measurements is no better than $10^{-29}\ s^{-3}$ and (ii) in this area pulsars move faster than anywhere, because here $\dot{\nu} \approx const$, $\ddot{\nu}$ changes its sign and hence the third derivative of ν has an extremum.

3. Non-monotonous variations of pulsar spin-down rate on large timescales

Variations of the pulsar rotational frequency may be complicated – periodic, quasi-periodic, or completely stochastic. Generally, it may be described as a superposition

$$\nu(t) = \nu_{ev}(t) + \delta\nu(t), \tag{2}$$

where $\nu_{ev}(t)$ describes the secular evolution of pulsar parameters and $\delta\nu(t)$ corresponds to the irregular variations. Similar expressions describe the evolution of $\dot{\nu}$ and $\ddot{\nu}$ after a differentiation. $\delta\ddot{\nu}(t)$ satisfies the obvious condition of zero mean value:

$$< \delta\ddot{\nu}(t) >_t = 0 \tag{3}$$

over the timespans larger than the characteristic timescale of the variations. The amplitude of the observed variations of $\ddot{\nu}$ is related to the dispersion of this process as

$$\sigma_{\delta\ddot{\nu}} = A_{\ddot{\nu}} = \sqrt{< (\delta\ddot{\nu})^2 >} \tag{4}$$

The second derivative values on the upper $\ddot{\nu}_+$ and lower $\ddot{\nu}_-$ branches in Fig. 1 may be approximately described as

$$\ddot{\nu}_{\pm}(t) = \ddot{\nu}_{ev}(t) \pm A_{\ddot{\nu}}(t) \tag{5}$$

for each pulsar. This equation describes an "average" pulsar, while the spread of points inside the branches reflects the variations of individual parameters over the pulsar ensemble and reaches 4 orders of magnitude.
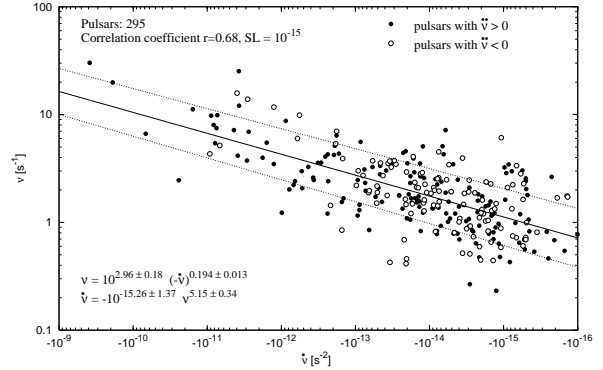

Fig. 4. The $\dot{\nu}-\nu$ diagram for the pulsars with the measured second derivative. The filled symbols are objects with positive $\ddot{\nu}$, the open ones – with negative $\ddot{\nu}$. The behaviour of both subsets is the same. The solid line represents the best fit corresponding to the braking index of $n \approx 5$, the dotted lines represent the 1-σ range.

The second derivative $\ddot{\nu}$ is the only parameter significantly influenced by the timing variations (see Section 4). Thus one can assume that the measured values of ν and $\dot{\nu}$ may be considered to be evolutionary, ν_{ev} and $\dot{\nu}_{ev}$ (since $\delta\nu$ and $\delta\dot{\nu}$ are small).

3.1. $\dot{\nu} - \nu$ diagram

Using the relations described above, the secular behaviour $\nu(t)$ (or, $\nu(\dot{\nu})$) may be found by plotting the studied pulsar group onto the $\dot{\nu}-\nu$ diagram (Fig. 4). The objects with $\ddot{\nu} > 0$ and $\ddot{\nu} < 0$ are marked as filled and open circles, correspondingly. It is easily seen that the behaviour of these two sub-groups is the same, which is in agreement with the smallness of the pulsar frequency variations in respect to the intrinsic scatter of $\nu(\dot{\nu})$. However, a strong correlation between ν and $\dot{\nu}$ ($r \approx 0.7$) is seen, and

$$\dot{\nu} = -C\nu^n, \tag{6}$$

where $C = 10^{-15.26\pm1.38}$ and $n = 5.15 \pm 0.34$. So, the secular evolution of an "average" pulsar goes according to the "standard" spin-down law with $n \approx 5$! This result is very interesting on its own, especially since the ν and $\dot{\nu}$ are always measured as independent values.

Note that the width of the fit on Fig. 4 is quite small – only about 1.5 orders of magnitude. If the spin-down is even approximately close to being described by the vacuum dipole model, then C should scale as $(B_0 \sin\chi)^2$, where B_0 is the polar field and χ is the magnetic inclination angle. But the range of $(B_0 \sin\chi)^2$ over the pulsar population is expected to

be of many orders of magnitude. Therefore, Fig. 4 once again shows that a simple vacuum dipole model is not adequate to observations.

It is important that there is no contradiction between the derived value of $n \approx 5$ and values of the braking index derived in the usual manner using $\ddot{\nu}$ (these values lie in the range from approximately -10^6 to 10^6). The measured ν and $\dot{\nu}$ are weakly affected by the timing noise, so the pulsars move along the $\dot{\nu} - \nu$ distribution mostly according to the secular component of $\ddot{\nu}$. At the same time the combination $\ddot{\nu}\nu/\dot{\nu}^2 = n$ includes both the secular and irregular components of $\ddot{\nu}$, which leads to anomalous braking indices.

The braking index $n \approx 5$ allows us to describe the pulsars spin-down as "quadrupole-like". But it is unlikely that such spin-down is due to the simple quadrupole structure of the NS magnetic field and the corresponding radiation. Indeed, in case of vacuum approximation, for the rotating quadrupole with magnetic field strength $B_0(R_{NS}) \sim 10^{12}$ G (where $R_{NS} \sim 10^6$ cm), the value of the parameter C from Eq. (6) would be as small as 10^{-26} (Manchester & Taylor, 1977; Krolik, 1991), which is ten orders of magnitude smaller than the values measured for the 295 and 1337 pulsars (see Figures 4 and 5). Moreover, the total contribution of all the high order multipole components (quadrupole etc.) is much smaller than that of the dipole component.

However, it is clear that the described simple quadrupole spin-down does not satisfy the modern concept of the pulsars' magnetospheres not being vacuum (Goldreich & Julian, 1969; Manchester & Taylor, 1977; Beskin et al., 1993).

A number of papers were published in the last years, where the possibility of quadrupole components of pulsars' magnetic fields was discussed (e.g. Zane & Turolla (2006)). Thus, at the same time, there is little reason to fully reject the hypothesis of a more complex pulsars' magnetic fields structure than only dipolar.

An additional argument in favour of the $\dot{\nu} - \nu$ correlation is, as has been stated above, the dependence between $\dot{\nu}$ and τ_{ch}, where the index of power is not equal to -1.0. Indeed, since $\tau_{ch} = -\frac{1}{2}\frac{\nu}{\dot{\nu}}$, and if the $\dot{\nu} - \nu$ correlation is absent, then ν may only bring in some scatter to the $\dot{\nu} - \tau_{ch}$ dependence without its slope changing (on a logarithmic scale). But, if $\dot{\nu}$ and ν are really correlated, and the slope (braking index) is about 5, the $\dot{\nu} - \tau_{ch}$ dependence should show a slope of about -1.2 which is roughly consistent with the measured value (see Eq. 1). Precisely,
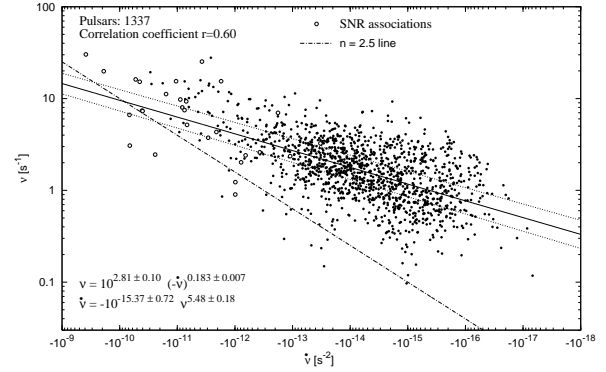

Fig. 5. The $\dot{\nu} - \nu$ diagram for the 1337 pulsars from the ATNF database. The distribution shows quite a strong correlation (r = 0.60). The slope of the distribution corresponds to $n = 5.48 \pm 0.18$. The slope of the dash-dot line on the plot corresponds to $n = 2.5$. It is clear that if the values of n for young pulsars are close to $2.5 \div 3.0$, then they should change to higher values as the pulsars evolve.

the slope of the $\dot{\nu} - \tau$ fit, α, and n are related by the equation:

$$n = \frac{\alpha}{\alpha - 1} \qquad (7)$$

So it clearly seen that n is strongly dependent on α. The value of n derived from $\alpha = 1.16 \pm 0.02$ is 7.25 ± 0.78. It differs from 5.15 ± 0.34 on a 2.5σ significance level. This value is less than the standard 3σ but close to it.

In a framework of pulsar spin-down analysis we ploted the $\dot{\nu} - \nu$ diagram for the 1337 pulsars (Fig. 5) taken from the ATNF pulsar database (Manchester et al., 2005). This subset does not include recycled, binary and anomalous pulsars, and satisfies the criteria described in section (2). The braking index derived from the plot is $n = 5.48 \pm 0.18$. This value is in a good agreement with the result described above. The precision of the slope measurement on Fig. 5 is so high mostly due to the large number of pulsars used.

The mean slope of the fit for the $\dot{\nu} - \nu$ distribution represents the average value of n for pulsars during their lifetime (≈ 5). Up to date there are several young pulsars with measured braking indices that are both accurate and precise. These are all smaller than but close to 3. The fact that the measured value of n was found to be greater than 3, could mean that pulsars do not evolve with a constant n (≤ 3).

Indeed, if the pulsars are really born with $n \leq 3$, and assuming they do not change this value during their evolution, then as they grow older, they should appear below the main distribution on the

$\dot\nu - \nu$ diagram: approximately in the area with $\dot\nu \sim -10^{-16} \div -10^{-15}\ s^{-2}$ and $\nu \sim 0.1\ Hz$.

As an example, the line with a slope which corresponds to $n = 2.5$ is shown on Fig. 5. This line significantly deviates from the main distribution in the area of the oldest pulsars. If the pulsars were to evolve along this line, they would arrive in the area below the main distribution as they aged.

Thus, the pulsars start their evolution with $n \leq 3$, which seems to be typical, and are likely to change their n to higher values as they evolve. Therefore, the authors consider to be reasonable the idea of young and old pulsars evolving with different values of n. These values are smaller for younger pulsars and larger for older ones.

Such changing of n can be explained, for example, by the changing of the dominating spin-down mechanism. In general, a number of scenarios were proposed, where pulsars' braking index would increase with time (see for example Cordes & Chernoff (1998) and references therein; see also Ruderman (2006)).

On the other hand, the increasing of the braking index may be explained by the evolution of the parameter C from Eq. (6). In case of a simple dipole spin-down $C \propto (B_0 \sin\chi)^2$ and it is not constant for pulsars during their lifetimes. The magnetic field decay and magnetic inclination angle evolution ($\dot\chi < 0$, see Davis & Goldstein (1970)) will lead to the decreasing of C for the old pulsars. Hence, the absolute values of $\dot\nu$ will be systematically smaller than the "unshifted" ones (when $C = const$). Thus the slope of the $\nu - \dot\nu$ distribution (braking index) will increase.

A similar situation also takes place in the model of electric current spin-down (Beskin et al., 1993). In this case, the relation $\dot\nu = -K\nu^3$ is also true, but $K \propto (B_0 \cos\chi)^2$ with $\dot\chi > 0$, instead of $(B_0 \sin\chi)^2$ with $\dot\chi < 0$.

In general, any pulsars' spin-down law should depend on the polar magnetic field B_0 and, very likely, on the magnetic inclination angle χ. Therefore, the idea described above should remain valid. If so, then the measured $n \approx 5$ means that pulsars' intrinsic braking index may be *less* than 5.

The provided analysis strongly suggests the presence of a $\dot\nu - \nu$ correlation for ordinary pulsars with a mean slope (braking index) close to 5. Younger pulsars seem to evolve with a lower n than the older ones. The pulsars' magnetic field may have a more complex structure than dipolar, however there is little reason to insist that $n \approx 5$ is due to a simple
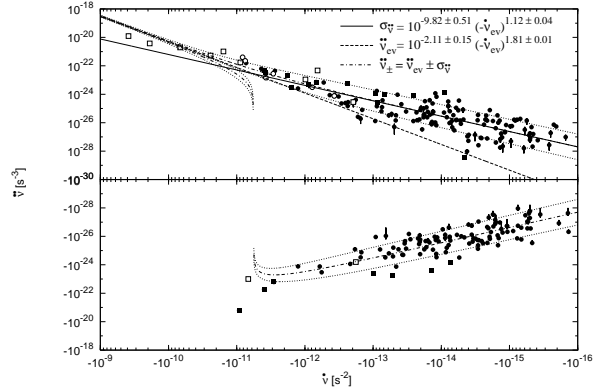

Fig. 6. The $\ddot\nu - \dot\nu$ diagram with the simple variations model. The solid line is the amplitude $A_{\ddot\nu} \equiv \sigma_{\ddot\nu}$ of the frequency second derivative variations, the dashed line is the secular term $\ddot\nu_{ev}$, and the dot-dashed lines are the envelopes of the oscillations $\ddot\nu_{ev} \pm \sigma_{\ddot\nu}$ with 1-σ ranges (dotted lines). Each pulsar spends the majority of its lifetime at or very near the envelopes. The pulsars are labeled as on Fig. 1

quadrupole spin-down. It may be a result of the pulsars' magnetic field decay and/or the evolution of the magnetic inclination angle.

3.2. *Some numerical results*

From Eq. (6) we may easily determine the relation between $\ddot\nu_{ev}$ and $\dot\nu$ as

$$\ddot\nu_{ev} = nC^{\frac{1}{n}}(-\dot\nu)^{2-\frac{1}{n}}, \tag{8}$$

which is shown in Fig. 6 as a thick dashed line. The same relation may be also estimated directly by using the asymmetry of the branches seen on Fig. 1 as

$$\ddot\nu(\dot\nu) = \frac{1}{2}(\ddot\nu_+ + \ddot\nu_-), \tag{9}$$

where $\ddot\nu_\pm$ are defined in Fig. 1. Such estimation, while being very noisy, is positive in the $-10^{-11} \div -10^{-15}\ \mathrm{s}^{-2}$ range and agrees quantitatively with the previous one.

The amplitude of the $\ddot\nu$ oscillations, $A_{\ddot\nu}$, may be easily computed in a similar way, by using $\ddot\nu_+$ and $\ddot\nu_-$, as $A_{\ddot\nu} = \frac{1}{2}(\ddot\nu_+ - \ddot\nu_-)$.

The behaviour of pulsars according to the derived relations is shown in Fig. 6. This simple variations model describes the observed branches, both positive and negative, rather well. We interpret the absence of negative branch objects with $\dot\nu < -10^{-11}\ \mathrm{s}^{-2}$, i.e. with $\tau_{ch} < 10^4$, as a prevalence of the second derivative's secular component over the varying one ($A_{\ddot\nu} < \ddot\nu_{ev}$) in this region. Older pulsars begin to change the sign of $\ddot\nu$ because of spin rate variations.
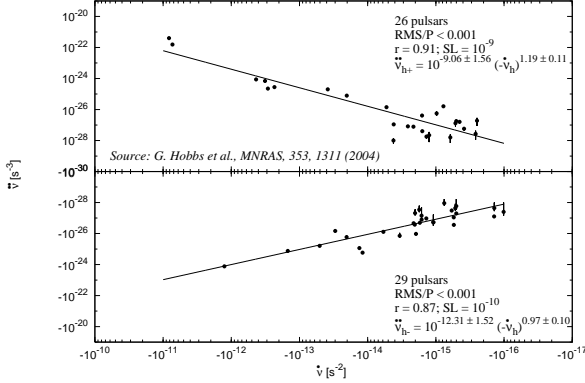
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Fig. 7. The $\ddot{\nu} - \dot{\nu}$ diagram for low noise pulsars. This diagram shows a possible difference between long timescale variations, discussed in the current work and the well-known "timing-noise", because some of the pulsars (especially with low $|\dot{\nu}|$) still have anomalous braking indices.
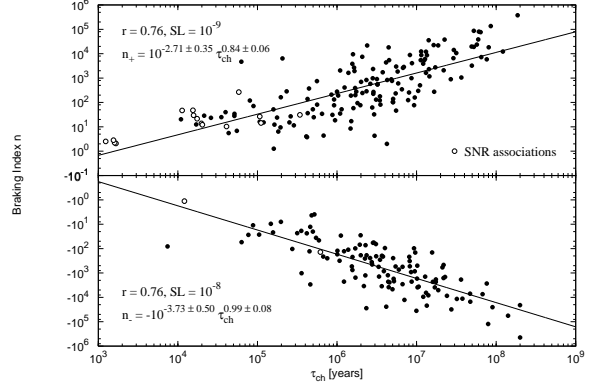

Fig. 8. The $n - \tau_{ch}$ diagram. The older pulsars have a larger spread of braking index values. But some of them show a very low timing noise within the timespan of observation (see Fig. 7).

4. Discussion and conclusions

In general, it is impossible to estimate the amplitudes of the frequency and its first derivative variations A_ν and $A_{\dot{\nu}}$ from the amplitude of the second derivative only (the knowledge of its complete power density spectrum is needed). However, if the spectral density is relatively localized and some characteristic timescale T of the variations exists, it is possible to set some limits on it. A rough estimation is $A_\nu \sim A_{\dot{\nu}} T$, $A_{\dot{\nu}} \sim A_{\ddot{\nu}} T$ and $A_\nu \sim A_{\ddot{\nu}} T^2$. On a long timescale the variations can not lead to pulsar spin-up, and therefore, the variations of frequency first derivatives are much smaller than their secular values, and $A_{\dot{\nu}} \sim A_{\ddot{\nu}} T \ll \dot{\nu}$, so $T \ll \dot{\nu}/A_{\ddot{\nu}}$. So, for the $-10^{-12} < \dot{\nu} < -10^{-15}$ s^{-2} range and corresponding values of $A_{\ddot{\nu}}$ from 10^{-23} s^{-3} to 10^{-26} s^{-3}, the characteristic timescale $T_{up} \sim 10^{11}$ s. Also, this characteristic timescale is obviously larger than the timespan of observations: $50 < T < 3 \cdot 10^3$ years. Assuming the constancy of T during the pulsar evolution and therefore the change of A_ν with time, we get $A_\nu \sim 10^{-3} \div 10^{-7}$ Hz. For such a model the pulsar frequency varies with the characteristic time of several hundred years and the amplitude from 10^{-3} Hz for young objects to 10^{-7} Hz for older ones.

The characteristic timescale T measured by the method above does not depend on the derived value of $n \approx 5$. Therefore, even if the braking index is significantly bigger than 5, the values of T and A_ν will not be changed.

The physical reasons of the discussed non-monotonous variations of the pulsar spin-down rate may be similar to the ones of the timing noise on a short timescale. Several processes had been proposed for their explanation (Cordes & Greenstein, 1981) – from the collective effects in the neutron star superfluid core to the electric current fluctuations in the pulsar magnetosphere. Whether these processes are able to produce long timescale variations is yet to be analysed. On a short timescale, the pulsars show different timing behaviour. But on the long timescale their behaviour seems to be alike.

The argument in favour of the similarity between the discussed variations and the timing noise is the coincidence of the timing noise $\ddot{\nu}$ amplitude extrapolated according to its power spectrum slope (Baykal et al., 1999) to the timescale of hundreds of years, with the $A_{\ddot{\nu}}$ derived from our analysis for the same $\dot{\nu}$, i.e. the same ages.

At the same time, there are several low noise pulsars with large or negative $\ddot{\nu}$ (see Figs. 7 and 8). For 55 pulsars studied in (Hobbs et al., 2004) the timing noise is nearly absent (RMS $< 1 \cdot 10^{-3}P$). Some of them have anomalous values of $\ddot{\nu}$, which are well consistent with the $|\ddot{\nu}|$ - $\dot{\nu}$ correlation (Cordes & Downs, 1985; Arzoumanian et al., 1994) and have a wide range of $\dot{\nu}$. This shows a possible difference between timing noise and the long timescale variations described above.

In any case, the principal point is that all the pulsars evolve with long-term variations, and the timescale of such variations significantly exceeds several tens of years. That explains the anomalous values of the observed $\ddot{\nu}$ and braking indices and gives reasonable values of the underlying secular spin-down parameters.

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